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APPENDIX B

CALCULATED SENSITIVITY OF AIRBORNE WEATHER RADARS

by

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and

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A study performed under the joint sponsorship of
Aeronautical Radio, Inc., and
The Air Transport Association of America.

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APPENDIX B

The performance of a weather radar may be considered under headings of resolution and sensitivity. Decreasing the wavelength would increase both these factors if it were not for an accompanying increase in attenuation by rain. The attenuation works against the enhanced resolution by introducing distortion, nearby precipitation patterns casting shadows on more distant patterns. It works powerfully against the enhanced sensitivity; at shorter wavelengths sensitivity depends more on the amount of intervening rain than it does on the distance away of the target rain. The present report will not deal with resolution, but will deal carefully with the matter of sensitivity.

The performance of a weather radar may be described by the equation

$$P_r = \frac{P_o\, h\, A \sum \sigma}{8 \pi r^2} \left(\frac{A_e}{A} F \right), \tag{1}$$

where P_r is the power received at the radar, P_o is the power transmitted, h is the pulse length, A is the geometrical area of the antenna, $\sum \sigma$ is the sum of the back-scatter cross-sections of all the precipitation particles in unit volume, and r is the range. A_e is the effective area of the antenna for extended targets and is less than A. The exact value of A_e is not easily determined. For this reason A only is retained in the equation and a conservative estimate (1/2) is made for A_e/A. The term F is a factor due to unknown causes which a careful experimental check by the M.I.T. Weather Radar Project has shown to be about 1/5. Therefore we have allowed a value of 0.10 for factors in the bracket. It is felt that with these factors, Eq. (1) represents the performance of a well-maintained radar as closely as present knowledge permits.

Equation (1) is valid, provided the beam is filled with scatterers and provided that attenuation due to intervening rain and atmospheric gases may be neglected.

The back-scatter cross section of a raindrop is given, according to Rayleigh's approximation, as

$$\sigma = \frac{\pi^5 D^6 |K|^2}{\lambda^4}, \tag{2}$$

where D is the drop diameter, $K = \frac{m^2 - 1}{m^2 + 2}$, m being the complex refractive index of water, and λ is the wavelength of the radar. Rayleigh's approximation is applicable (see Fig. 5x, Marshall, East and Gunn, 1952).

The sum of the sixth powers of the drop diameters in unit volume is given by

$$\sum D^6 = 2.0\, R^{1.6}\, 10^{-10}\ \mathrm{cm}^6\ \mathrm{cm}^{-3} \tag{3}$$

where R is the rate of rainfall in mm hr^{-1}. (This relation is also quoted from Marshall, et al (1952).) It depends on the size distribution of the drops, as obtained by Laws and Parsons (1943) and Marshall and Palmer (1948).

Combining Eqs. (1), (2) and (3), and solving for R one obtains:

$$R = \lambda^{2.50}\, r^{1.25}\, P_r^{0.625} \left[25\, \pi^4\, 10^{-13}\, A |K|^2\, P_o\right]^{-0.625}, \quad (4)$$

with all the quantities in c.g.s. units, and R in mm hr^{-1}. (Here A_e/AF has been put equal to 0.10 as discussed above.)

The minimum detectable rainfall R_m may be calculated from this equation if, for P_r, one uses the minimum detectable signal power. At the 1952 Radar Weather Conference such minimum values were reviewed (see Appendix I), and the consensus of opinion seemed to be that min. detectable P_r is about 4×10^{-13} watts for both 3.2 and 5.7 cm radar. This is the value of P_r that will be used throughout this report.

Putting $|K|^2 = 0.93$ (a value very nearly independent of wavelength) we rewrite Eq. (4) for the minimum detectable rainfall in more convenient units:

$$\underset{(\text{mm hr}^{-1})}{R_m} = 0.0981 \left(\frac{\lambda}{5.7 \text{ cm}}\right)^{2.50} \underset{(\text{mi})}{r^{1.25}} \left(\frac{P_r}{4\times10^{-13} \text{ w}}\right)^{0.625} \left(\frac{P_o}{40 \text{ kw}}\ \frac{h}{300\ \text{m}}\right)^{-0.625} \left(\frac{d}{18''}\right)^{-1.25}, \quad (5)$$

where d is the diameter of the (circular) antenna.

When the beam is not filled by precipitation at range r, the equation needs to be modified. The diameter of a circular beam is (between half-power points) $r\, 1.14\lambda/d$; hence, if we view a storm of linear dimension L, it follows that the beam will be just filled at range r_o, such that

$$r_o \frac{1.14\lambda}{d} = L. \quad (6)$$

Beyond r_o the beam will no longer be filled, and here P_r in Eq. (1) should be proportional to r^{-4} rather than to r^{-2}. Carrying through this modification, taking care that the two forms of Eq. (1) must agree at $r = r_o$, results in

$$R_m^1 = 0.00218 \left(\frac{\lambda}{5.7 \text{ cm}}\right)^{3.75} r^{2.50} \left(\frac{P_r}{4 \times 10^{-13} \text{ w}}\right)^{.625} \left(\frac{P_o}{40 \text{ kw}} \frac{h}{300 \text{ m}}\right)^{-.625} \left(\frac{d}{18''}\right)^{-2.50} \left(\frac{L}{3 \text{ mi}}\right)^{-1.25}. \qquad (7)$$

(mm hr^{-1}) $\qquad$ (mi)

Equations (5) and (7) give the minimum detectable rainfall without allowing for attenuation by intervening rain and by air and water vapor. Such attenuation causes a drop in the received power, and may be allowed for most easily by a corresponding drop in P_o, or an equivalent increase in $R_m^{1.6}$, in the above equations.

Attenuation by rain may be expressed accurately by

$$\underset{\text{(in db)}}{\text{radar attenuation by rain}} = K' \int_0^r R^{\alpha}\, dr, \qquad (8)$$

where

λ	3.2	5.7 cm
K'	0.0288	0.0094
α	1.3	1.1

Table I

provided R is in mm hr^{-1} and r in miles (data from Marshall, et al (1952)). The values of α are sufficiently close to unity to allow the simplification:

$$\underset{\text{(in db)}}{\text{radar attenuation by rain}} = K \times \underset{(\text{in mm hr}^{-1} \times \text{miles})}{\text{quantity of intervening rain}} \qquad (9)$$

where K is only slightly dependent on R. If **R** is about 30 mm hr^{-1}, K has the values:

λ	3.2	5.7 cm
K	0.08	0.013 db/(mm hr^{-1} mi)

Table 2

The attenuation by atmospheric gases (notably oxygen and water vapor) varies with temperature, pressure and humidity. Since the values are small, only representative figures, corresponding to a temperature just above 0°C, standard pressure, and water vapor pressure close to saturation, need be used. Thus, the radar attenuation by gases, k, may be taken to be

λ	3.2	5.7 cm
k	0.054	0.036 db mi^{-1}

Table 3

These values are taken largely from Ryde (1946).

There are, of course, a large number of ways in which the variation of the minimum detectable rainfall with the various radar parameters, with intervening attenuation and with the extent and intensity of the storm to be detected, may be displayed. One practical and useful way is to show the intervening rain y (in mm hr^{-1} x miles), through which a storm of given extent and intensity may be detected, as a function of range, r. Neglecting attenuation by the gases, this relationship may be derived as follows. (It is sketched in dashed lines in Fig. 1)

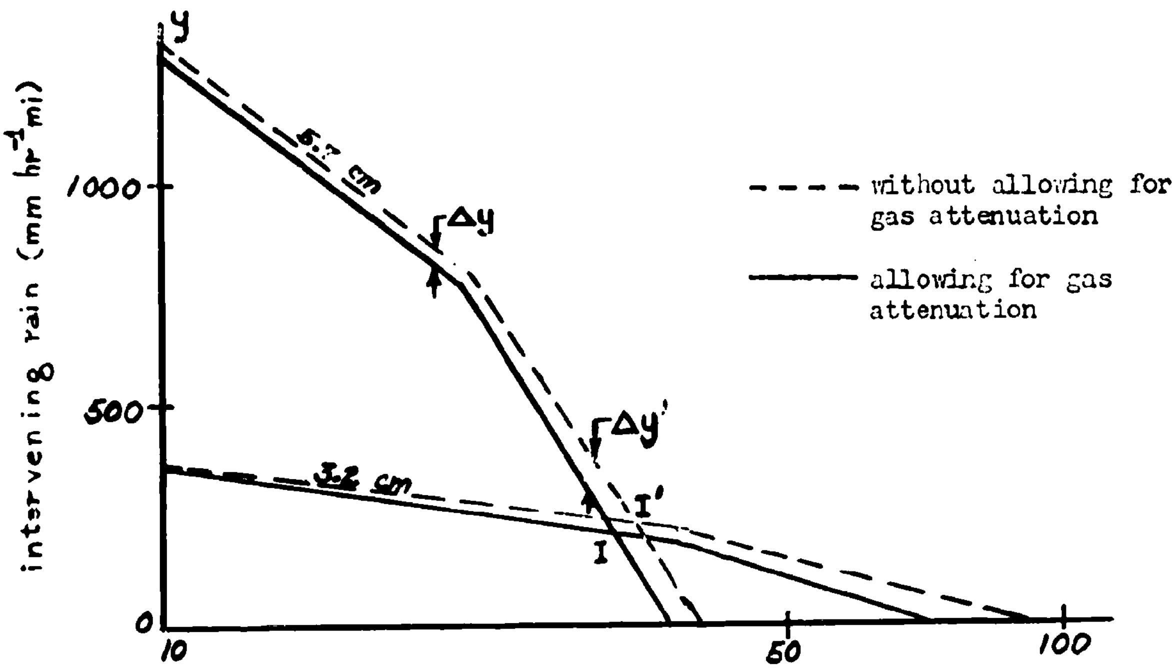


Fig. 1

The radar detects $R_m(r)$ (the minimum detectable rainfall at range r, as given by Eqs. (5) or (7)). If one is only interested in detecting R_o (a more intense rainfall), then the difference in power

$$10 \log \left(\frac{R_o}{R_m} \right)^{1.6} \text{ db}$$

may be used up by attenuation. Thus

$$Ky = 10 \log \left(\frac{R_o}{R_m} \right)^{1.6} . \tag{10}$$

As long as the beam is filled with precipitation, Eq. (5) applies for R_m, and

$$y = -\frac{20}{K} \log r + \frac{16}{K} \log \frac{R_o}{C} . \tag{11}$$

Beyond range r_o, the beam is no longer filled and instead of Eq. (11) one has (combining Eqs. (10) and (7)):

$$y' = -\frac{40}{K} \log r + \frac{16}{K} \log \frac{R_o}{C'} . \tag{12}$$

Here

$$C = \frac{R_m}{r^{1.25}} \quad \text{and } C' = \frac{R'_m}{r^{2.50}} , \tag{13}$$

are constants depending on the radar parameters, which may be obtained from Eqs. (5) and (7). Plots of y and y' against log r are straight lines.

Allowance for gas attenuation is easily made, simply by <u>reducing</u>, for value of r, the value of y (or y') by an amount

$$\Delta y = \Delta y' = \frac{kr}{K} , \tag{14}$$

This correction is quite small, so that in spite of it, the plots of y or y' vs. log r still appear to be nearly straight (the solid lines of Fig. 1 and all lines of Fig. 2).

The effect of wave length is emphasized here. Loci of y or y' are therefore drawn in pairs: one locus for 3.2 and one for 5.7 cm equipment, with all other parameters unchanged. A typical example of such a pair is sketched roughly in Fig. 1.

Of special interest in the point of intersection (I) of the two curves. The ordinate (y_0) of this point is the intervening rain at which the two wavelengths have the same range. For lower values of y, the shorter wavelength has the better range; for higher values of y, the longer wavelength is much to be preferred. Neglecting gas attenuation, y_0 turns out to be independent of all radar parameters and of $\dot{R}_0$ and L, provided the beams are either both filled or both unfilled. If the 3.2 cm beam is filled, but the 5.7 cm beam is not (a frequent case), y_0 turns out to depend somewhat on transmitter power, pulse length, etc., but not on the antenna size. The effect on y_0 of the attenuation by the gases is generally unimportant.

Figure 2 shows the final results of the calculations in the form outlined above; construction lines have been omitted and three pairs of loci are given, each pair having one member of 3.2 cm and one for 5.7 cm. This chart may best be discussed in terms of a particular pair of loci: the solid lines, for instance, refer to transmitter power 120 kw, antenna aperture 18 inches in diameter, pulse length 300 meters, intensity of target rainfall 10 mm/hr. For amounts of intervening rain up to 200 mm hr^{-1} miles, the 3.2-cm equipment gives greater ranges than the 5.7 cm. With this much intervening rain, both wavelengths have a limiting range of 31 miles. As one proceeds to greater amounts of intervening rain, the range at 3.2 cm continues to drop off very rapidly, so that with twice this much intervening rain the range is about 7 miles. At 5.7 cm, on the other hand, the same 400 mm/hr of rain drops the range back by less than 15 per cent to 27 miles. It was necessary to specify the extent of the target shower, since this determines whether or not the target precipitation completely fills the cross section of the radar beam. A shower of linear dimensions 3 miles high by 3 miles wide was chosen rather arbitrarily for all curves. At 5.7 cm this target fills the beam of the 18" antenna at ranges up to 21 miles, the narrower 3.2 cm beam at ranges to 38 miles. Kinks in the loci under discussion may be noted at those ranges.

The amount of intervening rain at which the two wavelengths have the same range, i.e., about 200 mm hr^{-1} mi, is not very much in stormy weather situations, and so attenuation at 3.2 cm is seen to be a very serious difficulty. It is not just a matter of the range being limited, there is the uncertainty whether one sees light rain through a small amount of intervening precipitation or heavy target rain through much intervening precipitation.

Let us now look at the alternatives. Reduction of the power by a factor 3 or of the minimum detectable target rainfall to 5 mm hr^{-1} makes for a serious further reduction in range (dotted lines). On the other hand, the effect of increasing the size of dish is to extend the range very considerably (broken lines). The range at the longer wavelength comes between 57 miles in the clear, and 45 miles with about 400 intervening mm hr^{-1} miles. Effectively the same result can be achieved by increasing the target rainfall to 50 mm hr^{-1}.

The chart shows three pairs of curves covering four sets of parameters at the two wavelengths. It is, however, easy to construct further loci by moving the lines, without changing their direction, in the following way. (Such an extension of the chart is only approximate, as this procedure should really be applied to the construction lines of Fig. 1.)

To allow for variation in

(a) transmitter power P_o, move curves vertically, changing the height of the kink (or any other point of fixed range) by

$$y' - y = \frac{1}{K} \text{ x (change in } P_o \text{ in db)}$$

$$(= \frac{10}{K} \log P_o'/P_o),$$

(b) pulse length h, proceed exactly as for variation in P_o,

(c) target rainfall R_o, move curves vertically, changing the height of the kink, etc., by

$$y' - y = \frac{1.6}{K} \text{ x (change in } R_o \text{ in db)}$$

$$(= \frac{16}{K} \log R_o'/Ro),$$

(d) antenna diameter d, move curves horizontally, adjusting the range of the kink proportionately to d, (i.e. $r_o'/r_o = d'/d$);

(e) linear dimension of target rain, L, move kink along left part of locus (produced, if necessary) to a range proportional to L; (i.e. $r_o'/r_o = L'/L$). Then draw right part of locus parallel to the right part of the locus initially given.

It is concluded from this investigation that attenuation by intervening rainfall effects a very serious limitation to the sensitivity of 3 cm weather radar. A frontal line of showers is likely to involve the passage of the radar beam through a few hundred mm hr^{-1} miles of rain, probably, though not certainly, less than 500. Sensitivity at 3 cm varies very rapidly with the amount of intervening rain, and this rapid variation in itself would lead to uncertainty in interpreting signals. Five hundred mm hr^{-1} miles is enough to render 3 cm equipment practically inoperative.

At wavelength 5.7 cm, attenuation is appreciable but the sensitivity changes more gradually with intervening rain, and the range would never be reduced by more than 30 per cent. (This assumes that the shower does not fill the beam; in all likelihood this will be the case at limiting ranges with the relativeity small size of an airborne antenna.) At this wavelength, however, it will be difficult to achieve the maximum desired ranges even in clear air.

The most promising solution would appear to be the use of wavelength 5.7 cm, doing everything possible in design and construction to keep up the sensitivity. At the same time, it may be reassuring to note that the range increases with the intensity of target rainfall. The range is doubled in going from the 10 mm hr^{-1} specified here to 60 mm hr^{-1}. Radar observations on thundershowers almost always indicate intensities of this order at the core, or at any rate a core condition providing a signal equivalent to 60 mm hr^{-1} in radar reflectivity.

The two wavelengths have been compared only in sensitivity. For a constant size of antenna, they will also differ in resolution: the shorter wavelength will have a correspondingly narrower beam and correspondingly higher resolution. But attenuation will introduce distortion into the pictures, altering outlines and casting shadows of nearer showers on more distant ones. It may be that the initially better resolution of the shorter wavelength will be more than cancelled out by attenuation, in the same way as the initially higher sensitivity. The situation with regard to resolution and distortion has not been studied; the best reference to date would appear to be Atlas and Banks (1951).

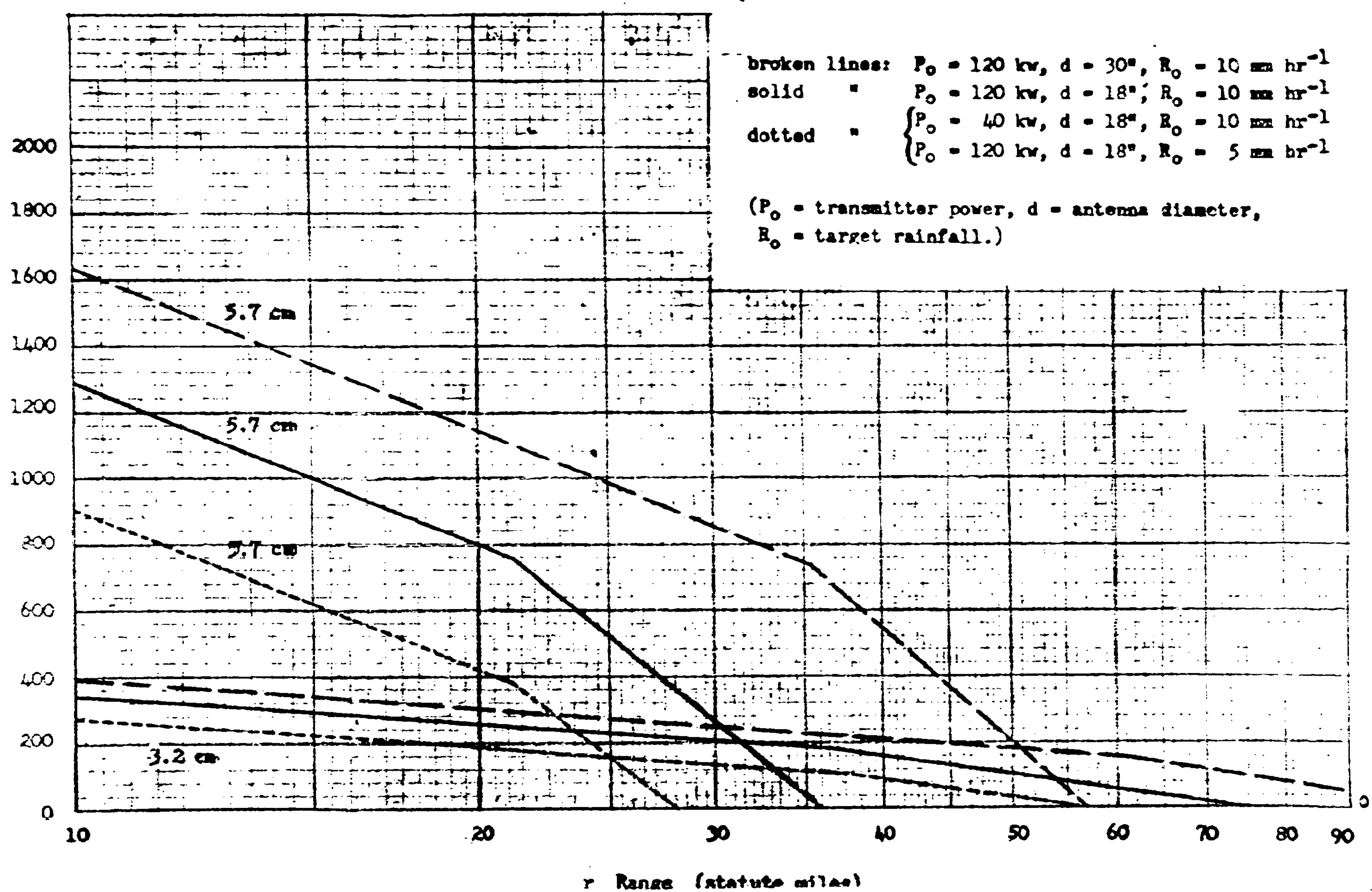
TABLE 4

Performance of Radars of Different Wavelengths

This table is taken from a brief note (pages B-37/38) in the Proceedings of the Third Radar Weather Conference, held at McGill University, 17-19 September 1952. The data represent the general consensus of the Conference, which comprised the majority of radar weather specialists.

λ (cm)	0.9	1.25	3.2	5.6	10.0
Maximum available transmitter power (P_o, kw)	20	40	350	300	1000
Minimum receivable signal power (P_r, watts)	10^{-12}	5×10^{-13}	4×10^{-13}	3×10^{-13}	2×10^{-13}
Performance factor ($P_o/P_r \times \lambda^{-4}$, relative to 10 cm equipment)	61	65	17	2.04	1
Performance level (db)	18	18	12	3	0
Attenuation (two-way transmission)					
Cloud (density 1 gm m^{-3}) (db mi^{-1})	3.2	1.75	0.28		
Rain - db ($mm\ hr^{-1} \times mi)^{-1}$ (at R = 100 mm hr^{-1})			0.09	0.015	0.00

Figure 2


broken lines: P_o = 120 kw, d = 30", R_o = 10 mm hr^{-1}
solid " P_o = 120 kw, d = 18"; R_o = 10 mm hr^{-1}
dotted " P_o = 40 kw, d = 18", R_o = 10 mm hr^{-1}
P_o = 120 kw, d = 18", R_o = 5 mm hr^{-1}
(P_o = transmitter power, d = antenna diameter, R_o = target rainfall.)
5.7 cm
5.7 cm
5.7 cm
3.2 cm
2000
1800
1600
1400
1200
1000
800
600
400
200
0
10
20
30
40
50
60
70
80
90
r Range (statute miles)

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Ryde, J. W., 1946: Contribution in "Meteorological Factors in Radio Wave Propagation." The Physical Society, London.

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